

Intraday optical variability of BL Lacertae

Nankun Meng,¹ Jianghua Wu,^{1★} James R. Webb,² Xiaoyuan Zhang¹ and Yan Dai¹

¹Department of Astronomy, Beijing Normal University, 100875 Beijing, China

²Department of Physics, Florida International University, Miami, FL 33199, USA

Accepted 2017 April 27. Received 2017 April 27; in original form 2016 December 19

ABSTRACT

We monitored BL Lacertae simultaneously in the optical *B*, *V*, *R* and *I* bands for 13 nights during the period 2012–2016. The variations were well correlated in all bands and the source showed significant intraday variability (IDV). We also studied its optical flux and colour behaviour, and searched for inter-band time lags. A strong bluer-when-brighter chromatism was found on the intra-night time-scale. The spectral changes are not sensitive to the host galaxy contribution. Cross-correlation analysis revealed possible time delay of about 10 min between variations in the *V* and *R* bands. We interpreted the observed flares in terms of the model consisting of individual synchrotron pulses.

Key words: galaxies: active – BL Lacertae objects: individual: BL Lacertae – galaxies: photometry.

1 INTRODUCTION

BL Lacertae is the archetype of BL Lac class, which together with flat spectrum radio quasars, constitutes a violently variable class of active galactic nuclei known as blazars. Blazars are characterized by high and variable polarization, synchrotron emission from relativistic jets, core-dominated radio morphology and intense flux and spectral variability in all wavelengths ranging from radio to γ -ray on a wide variety of time-scales (Urry & Padovani 1995; Wagner & Witzel 1995; Böttcher et al. 2003). Blazar variations can be divided into long-term variability, short-term variability and microvariability. Magnitude changes of few hundredth to tenths over a day are called intraday variability (IDV) or microvariability (Miller, Carini & Goodrich 1989; Howard et al. 2004; Agarwal & Gupta 2015).

BL Lacertae, located at redshift value of $z = 0.0668 \pm 0.0002$ (Miller & Hawley 1977), is hosted by a giant elliptical galaxy with $R = 15.5$ (Scarpa et al. 2000). As its first spectral component peaks in near-IR/optical region, BL Lacertae is a typical low-frequency peaked blazar with the radio to X-ray spectral index equal to 0.84 (Fossati et al. 1998; Fiorucci, Ciprini & Tosti 2004).

BL Lacertae was continuously observed by several multiwavelength campaigns carried out by the Whole Earth Blazar Telescope (WEBT/GASP) (Villata et al. 2003, 2004; Bach et al. 2006; Raiteri et al. 2009). Numerous investigations have been carried out to search for the flux variations, spectral changes and periodicities (Epstein et al. 1972; Carini et al. 1992; Villata et al. 2002; Papadakis et al. 2003; Agarwal & Gupta 2015; Gaur et al. 2015). The majority of the observations revealed that its IDV amplitude is larger at higher frequencies and decreased as the flux increased. These studies confirmed the presence of bluer-when-brighter (BWB) trend,

but yielded no evidence for periodicities (Carini et al. 1992; Villata et al. 2002; Gaur et al. 2015). Hagen-Thorn et al. (2002) proposed the evidence for periodicity of 308 d for total flux variations in 22 yr. Most authors did not find any significant time lags between different optical bands during a single night observation (Nesci et al. 1998). However, Papadakis et al. (2003) found that the delay between *B*- and *I*-band light curves was ~ 0.4 h. In addition, a possible time lag between *e* and *m* bands was reported as ~ 11.6 min by Hu et al. (2006).

Polarimetric monitoring can offer information about the physical processes in blazar. Unlike radio polarization, optical polarimetry probes the central nuclear regions of blazar jets (Falomo, Pian & Treves 2014). The optical polarized emission, as a potential good tracer of the high-energy emission, is being widely studied (Marscher et al. 2008; Raiteri et al. 2013; Sorcia et al. 2013; Covino et al. 2015). Since blazar optical radiation is dominated by the synchrotron mechanism, this implies the presence of highly ordered large-scale magnetic fields (Westfold 1959). Observation of optical polarizations of BL Lac objects has shown the degree of polarization varies on time-scales from IDV to years (Impey et al. 2000; Tommasi et al. 2001; Sasada et al. 2008). Occasionally, the correlation (Hagen-Thorn et al. 2008) and anticorrelation (Gaur et al. 2014) between the total and polarized fluxes is observed, while in general no clear relation is found.

In this paper, we studied the IDV and colour behaviour of this object, and searched for the inter-band time lags with data collected during the period 2012–2016 with high temporal resolution. The results can give us insight into theoretical causes of variability. This paper is organized as follows: Section 2 describes the observations and data reductions, and Section 3 provides a brief introduction to various analysis techniques, followed by the results in Section 4. The discussion and conclusions are in Sections 5 and 6, respectively.

★ E-mail: jhwu@bnu.edu.cn

Table 1. Parameters of three telescopes.

Telescope	CCD resolution	CCD view
60 cm reflector	1.06 arcsec pixel ⁻¹	11 arcmin × 11 arcmin
85 cm reflector	1 arcsec pixel ⁻¹	33 arcmin × 33 arcmin
216 cm reflector	0.305 arcsec pixel ⁻¹	6.5 arcmin × 5.8 arcmin

2 OBSERVATIONS AND DATA REDUCTIONS

The monitoring was performed with three telescopes at Xing-long Station of the National Astronomical Observatories Chinese Academy of Sciences (NAOC). The details of the telescopes are shown in Table 1. We observed BL Lacertae in the *B*, *V*, *R* and *I* bands for 13 nights covering the period from 2012 November 5 to 2016 January 17. During 12 of those nights, we observed simultaneously in different bands, providing a total of 24 intraday light curves. The longest individual duration of observation was about 7.7 h. The entire observation log together with all the available results is presented in Table 2. More than 10 000 original data points were collected on the 13 nights.

The data reduction procedures included bias subtraction, flat-fielding, extraction of instrumental magnitudes and flux calibration. The pre-processing of the raw data was accomplished by using standard procedures in the IRAF¹ software. For each night, photometry was carried out with five different aperture radii, i.e. $\sim 1 \times \text{FWHM}$, $1.5 \times \text{FWHM}$, $2 \times \text{FWHM}$, $3 \times \text{FWHM}$, $4 \times \text{FWHM}$. The minimum standard deviation of photometric error corresponds to the best aperture and we finally selected the best aperture data for our analysis. Three local comparison stars (B, C, H in Fig. 1) were observed in the same field. The standard magnitudes of these stars in the *B*, *V*, *R* and *I* bands are given by Smith et al. (1985). The brightness of BL Lacertae was calibrated relative to the average brightness of stars B and C. They have similar magnitude and colour to BL Lacertae. Star H acted as a check star.

The host galaxy of BL Lacertae is relatively bright and its contribution to the magnitudes was subtracted after flux calibration in order to avoid contamination. Hyvönen et al. (2007) derived a *B* magnitude of 17.35 for the host galaxy of BL Lacertae. The host galaxy contribution in *V*, *R* and *I* bands was inferred by adopting the elliptical galaxy colours of $B - V = 0.99$, $V - R = 0.59$ and $V - I = 1.22$ from Mannucci et al. (2001). Villata et al. (2002) estimate that the host galaxy contribution to the observed flux is about 60 per cent of the whole galaxy flux. The magnitudes were transformed into flux and the host galaxy contribution was removed.

Given that short exposure times cause the data dispersion, a smoothing algorithm was implemented for the data in 2013, 2014 and 2015. The data were smoothed using the average of each 3 min bin. The overall light curves of this source are displayed in Fig. 2. Small black dots denote original data and different colour dots denote smoothed data in different passbands. There are about 2000 smoothed data points used for further calculation. For clarity, several band light curves are shifted correspondingly.

3 VARIABILITY DETECTION CRITERIA

To quantify the IDV of BL Lacertae, two statistical analysis techniques were adopted, the χ^2 test and ANOVA test.

¹ IRAF is distributed by the National Optical Astronomy Observatories, which are operated by the Association of Universities for Research in Astronomy, Inc., under cooperative agreement with the National Science Foundation.

3.1 χ^2 test

The χ^2 statistic is defined as

$$\chi^2 = \sum_{i=1}^N \frac{(V_i - \bar{V})^2}{\sigma_i^2}, \quad (1)$$

where $\bar{V}$ is the mean magnitude of all *i*th observation V_i with a corresponding error σ_i . The exact errors from the IRAF reduction package are smaller than the real ones by a factor of 1.3–1.75 (Gupta et al. 2008; Agarwal & Gupta 2015). We chose the factor of 1.5 for data processing to get a better estimate of the actual photometric errors. If the actual variability is greater than the critical value at the $N - 1$ degree of freedom and selected significance level, then the presence of variability can be claimed.

3.2 ANOVA test

de Diego et al. (1998) used the one-way ANOVA to investigate the variability of quasars. The mathematical description of the one-way ANOVA test is as follows: if y_{ij} represents the *i*th (with $i = 1, 2, \dots, n_j$) observation on the *j*th (with $j = 1, 2, \dots, k$) group, the linear model describing every observation is

$$y_{ij} = \bar{y} + g_i + \varepsilon_{ij}, \quad (2)$$

where $\bar{y}$ represents the mean value of the whole data set, $g_j = \bar{y}_j - \bar{y}$ the between-groups deviation and $\varepsilon_{ij} = y_{ij} - \bar{y}_j$ the within-groups deviation. Our observations on each night were divided into groups of five consecutive data points. The total sample variation can be separated between and within group deviations:

$$\sum_{j=1}^k \sum_{i=1}^{n_j} (y_{ij} - \bar{y})^2 = \sum_{j=1}^k (y_j - \bar{y})^2 + \sum_{j=1}^k \sum_{i=1}^{n_j} (y_{ij} - \bar{y}_j)^2. \quad (3)$$

Equation (3) can be shortened to $SS_T = SS_G + SS_R$. SS_T stands for the total sum of squares that describes the total deviation of the data with respect to the mean. SS_G stands for the right-hand side of equation (3) and SS_R is the total error.

The statistics correspond to the *F* distribution with $k - 1$ and $N - k$ degrees of freedom.

$$F = \frac{SS_G/(k - 1)}{SS_R/(N - k)}. \quad (4)$$

For a certain significance level, if *F* value exceeds the critical value, the null hypothesis will be rejected and implying the existence of variability.

4 RESULTS

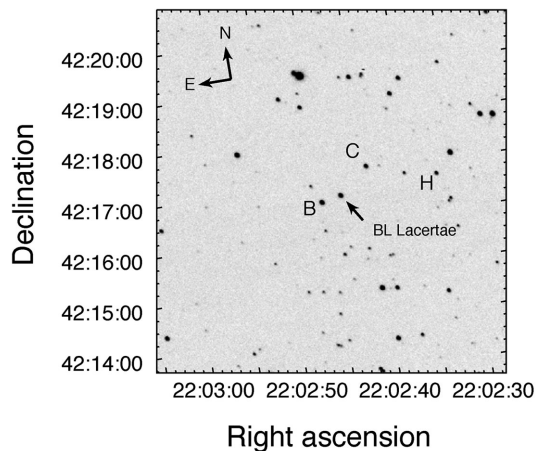
4.1 Light curves and flux variations

The light curves we obtained are shown in Fig. 2. We adopted two statistical analysis techniques described in Section 3 to search for variability and the results are listed in Table 2. The first column of Table 2 shows the observation date, while the second column records the telescope. The band and number of the data points are given in the third and fourth columns, respectively. The observation durations are listed in the fifth column and the test results of χ^2 and ANOVA are in the sixth and seventh columns. If *F* value exceeds the critical value (CV) at the 99 per cent significant level, the null hypothesis that there are no variations will be rejected. BL Lacertae is marked as Y if the variability conditions for each test are satisfied, while N means no variations. The final column is IDV amplitude.

Table 2. Results of BL Lacertae.

Date (yyyy mm dd)	Telescope	Band	No. of data points	Durations (h)	χ^2 test			ANOVA test			A (per cent)
					<i>F</i>	CV	Var?	<i>F</i>	CV	Var?	
2012 11 05	216 cm	<i>B</i>	77	4.960	2884	107.6	Y	26.26	3.012	Y	12.12
		<i>R</i>	77	4.954	3821	107.6	Y	37.96	3.012	Y	10.77
2013 10 19	85 cm	<i>V</i>	60	3.698	76.65	87.17	N	2.471	3.536	N	
		<i>R</i>	59	3.946	502.3	85.95	Y	0.674	3.791	N	
2013 10 20	85 cm	<i>V</i>	119	7.354	509.4	156.7	Y	3.268	2.358	Y	12.73
		<i>R</i>	119	7.397	453.4	156.7	Y	2.707	2.358	Y	10.42
2013 10 21	85 cm	<i>V</i>	36	2.199	40.41	57.34	N	2.645	5.944	N	
		<i>R</i>	36	2.186	37.59	57.34	N	3.022	5.944	N	
2014 11 01	60 cm 85 cm	<i>B</i>	14	0.802	4.407	27.69	N	1.103	99.40	N	
		<i>R</i>	10	0.559	0.830	21.66	N	0.253	99.40	N	
2014 11 02	60 cm 85 cm	<i>B</i>	97	5.981	11.54	131.1	N	0.428	2.608	N	
		<i>R</i>	94	5.798	17.97	127.6	N	0.684	2.690	N	
2014 11 03	60 cm 85 cm	<i>B</i>	101	6.230	22.45	135.8	N	1.517	2.535	N	
		<i>R</i>	81	4.968	32.67	112.3	N	5.749	2.889	Y	
2015 10 16	60 cm	<i>I</i>	123	7.589	40.93	161.3	N	6.820	2.310	Y	
2015 10 17	60 cm 85 cm	<i>V</i>	45	2.742	6.795	68.71	N	1.889	4.539	N	
		<i>I</i>	83	5.094	50.11	114.7	N	10.58	2.889	Y	
2015 10 18 ₁	60 cm 85 cm	<i>B</i>	29	1.740	45.90	48.28	N	4.102	9.449	N	
		<i>I</i>	36	2.162	9.297	57.34	N	1.846	5.944	N	
2015 10 18 ₂	60 cm 85 cm	<i>B</i>	82	3.568	91.27	113.5	N	1.400	2.889	N	
		<i>I</i>	85	3.665	14.65	117.1	N	0.320	2.783	N	
2015 10 19	60 cm 85 cm	<i>B</i>	116	7.171	1603	153.2	Y	134.6	2.358	Y	15.85
		<i>I</i>	125	7.722	771.4	163.6	Y	135.0	2.266	Y	13.49
2016 01 16	85 cm	<i>V</i>	16	0.565	15.11	30.58	N	0.023	26.87	N	
		<i>R</i>	15	0.528	21.03	29.14	N	0.365	26.87	N	
2016 01 17	85 cm	<i>V</i>	40	1.468	319.5	62.43	Y	5.732	5.116	Y	6.72
		<i>R</i>	40	1.469	349.6	62.43	Y	6.079	5.116	Y	5.31

Note. We displayed two separate results (2015 10 18₁ and 2015 10 18₂) because of the existence of gaps in the light curves on 2015 October 18.

**Figure 1.** Finding chart of BL Lacertae in the *B* band on 2015 October 19.

The IDV amplitudes are given by Heidt & Wagner (1996):

$$A = \sqrt{(m_{\max} - m_{\min})^2 - 2\sigma^2}, \quad (5)$$

where $m_{\max}$ and $m_{\min}$ are the maximum and minimum magnitudes, and σ is the standard deviation.

The variations were well correlated in all bands. As can be seen, there are variations on four nights that are detected by both tests. We calculated their amplitudes using the above equation. The maximum amplitude of IDV is 15.85 per cent in the *B* band on 2015 October 19. The amplitude of IDV is greater in higher energy bands. This has been observed in BL Lacertae (Nesci et al. 1998; Webb et al. 1998; Nikolashvili & Kurtanidze 2004). The amplitude of variability in

different bands is changed during different nights. The comparison results of two tests indicate that ANOVA shows efficient detection of IDV with small flares. Several small flares can be seen in the light curves on four of these nights, which can be claimed as the IDV by ANOVA test. We interpreted the observed flares in terms of the model consisting of individual synchrotron pulses in another section below.

4.2 Colour behaviour

We investigated the colour behaviour with respect to the brightness of BL Lacertae for each separate night. The colour indices of $B - R$, $V - R$, $V - I$ and $B - I$ are calculated by using the almost simultaneous *B*, *V*, *R* and *I* magnitudes. Because of the existence of gaps in the light curves on 2015 October 18, we plotted two separate diagrams and the results are displayed in Fig. 3. We fitted the colour–magnitude diagrams with a linear model (where r is the correlation coefficient) and calculated the Spearman correlation coefficient (Sc). Three of them were left out since they were not able to meet the conditions of the *F*-test after linear regression analysis.

The source exhibits a BWB trend that has been reported previously (Racine 1970; Speziali & Natali 1998; Vagnetti, Trevese & Nesci 2003; Stalin et al. 2006; Papadakis, Villata & Raiteri 2007; Ikejiri et al. 2011; Gaur et al. 2015; Wiercholska et al. 2015) and unambiguously confirmed as a universal aspect in blazars by Ikejiri et al. (2011). Especially on 2015 October 19, the Sc value reaches up to 0.9689, indicating the significant linear correlation between colour index and magnitude. Too few data points on several observations, particularly on 2016 January 16, made the fitting unreliable. The Sc values are less than 0.6 on three nights that all have flares.

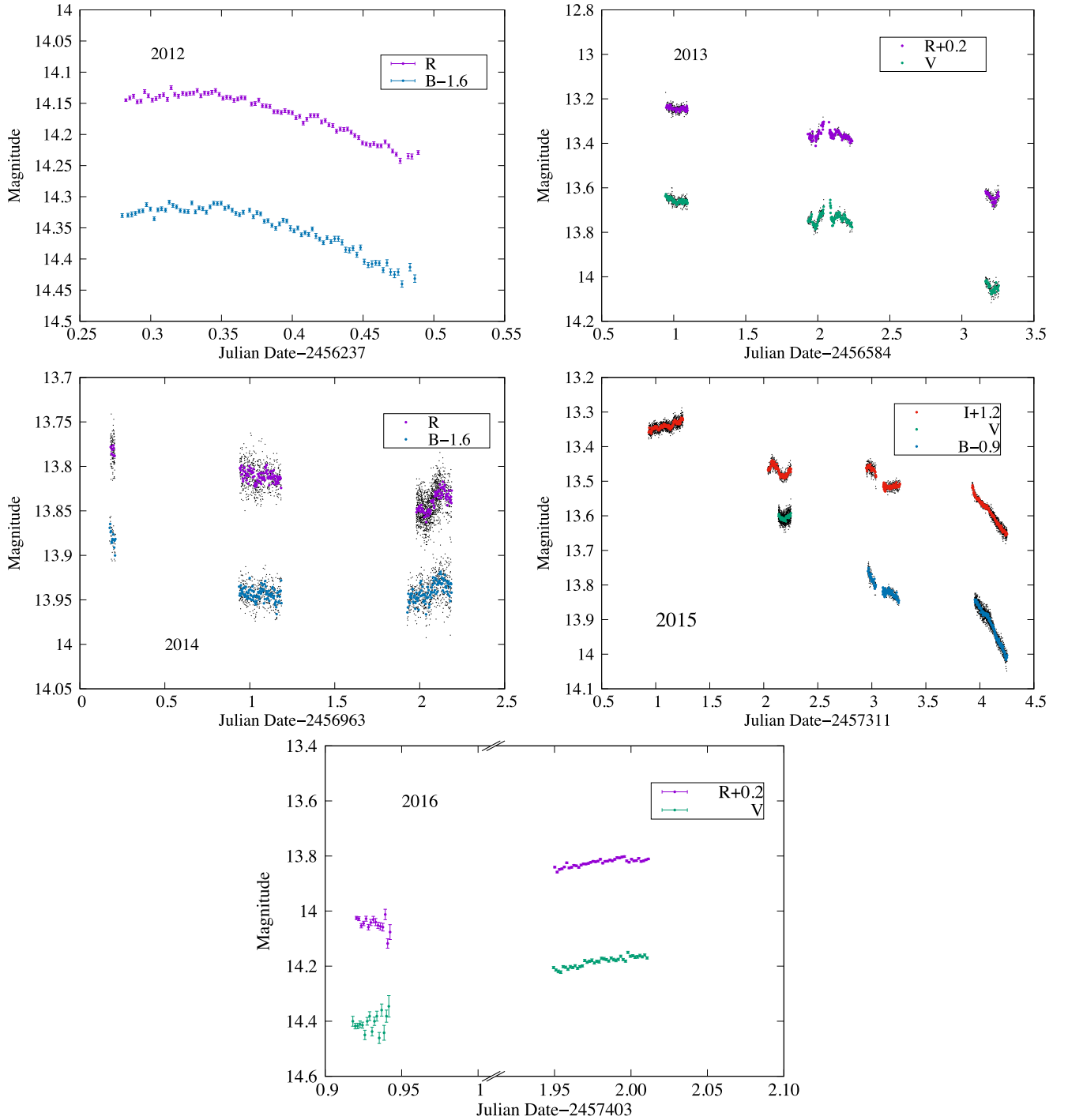


Figure 2. Light curves of BL Lacertae in the B , V , R and I bands in 5 yr. Small black dots denote original data. Different colour dots denote smoothed data in different passbands. For clarity, several band light curves are shifted correspondingly.

The existence of flares may have the opposite effect on the colour behaviour of BL Lacertae. Gaur et al. (2015) interpreted the weakening of the colour–magnitude correlations as the superposition of many distinct new variable components. Webb (2016) proposed a new definition of microvariability as the short-term oscillations distinct from the linear variability. Wiercholska et al. (2015) argued that the observed BWB behaviour was intrinsic to the jet emission regions.

The colour trend in blazars on intraday time-scales can help us investigate the origin of blazar emissions. Fig. 4 shows the plots of each colour index against intraday time-scale with fitting lines. The best-fitting values of slope, intercept, r , S_c and null hypothesis probability are listed in Table 3. A positive slope implies significant positive correlation between colour index and time when both r and S_c are greater than 0.9. We also gave the F -test results and the critical values after linear regression analysis. Since some of them

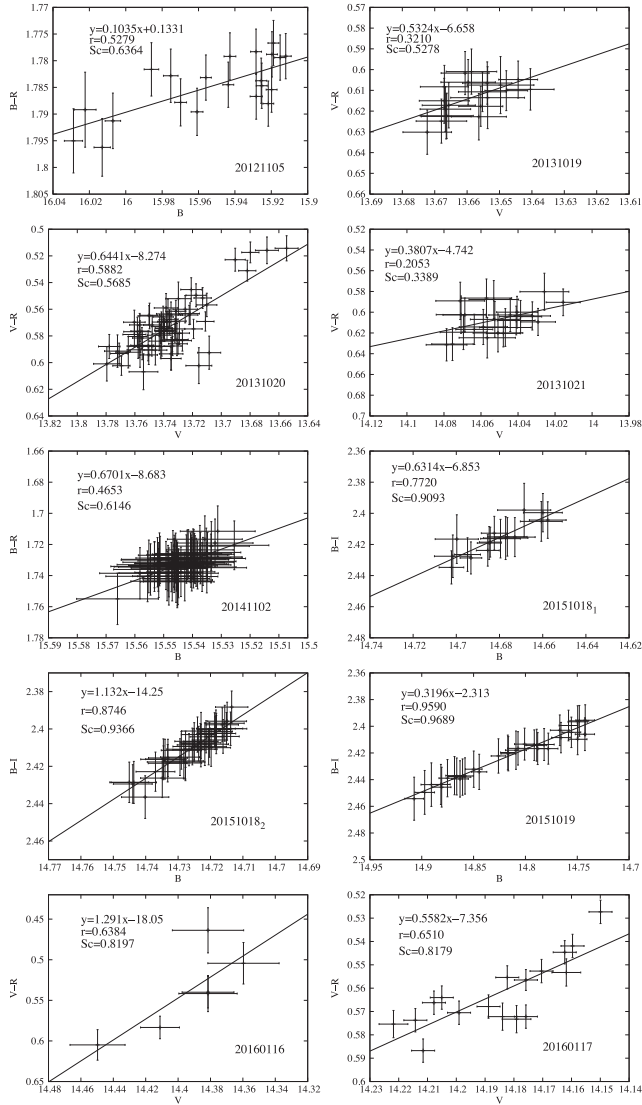


Figure 3. Colour-magnitude diagrams of BL Lacertae.

were not able to meet the conditions of the F -test, we did not plot them in Fig. 4. The average colours $\langle CI \rangle$ and the corresponding spectral indices $\langle SI \rangle$ are given in Table 3 as well. According to Wiercholska et al. (2015), the average spectral indices are derived simply as

$$\langle \alpha_{AB} \rangle = \frac{0.4 \langle A - B \rangle}{\log(\nu_A / \nu_B)}, \quad (6)$$

where A and B stand for different bands, and ν_A and ν_B are effective frequencies of the respective bands (Bessell, Castelli & Plez 1998). The spectral indices varied slightly as shown in Fig. 5. The spectral indices of α_{BR} , α_{BI} and α_{VR} changed by only 0.21, 0.02 and 0.45, respectively. The α_{VI} even remained unchanged due at least partly to the few data points. The accretion disc radiation is expected to be overwhelmed by that from the strongly Doppler-boosted jets, so the observed spectral variations in blazars cannot be explained by the accretion disc. The relatively steep spectral indices indicate strong synchrotron emission from the blazar jet and small accretion disc contribution (Agarwal et al. 2016).

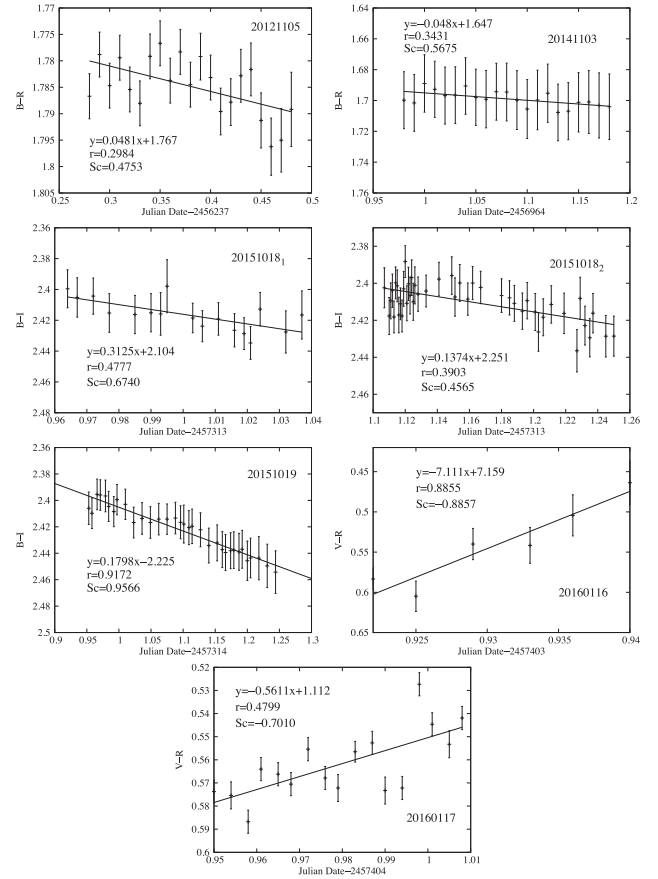


Figure 4. Colour index versus intraday time-scale.

4.3 Cross-correlation analysis and time lags

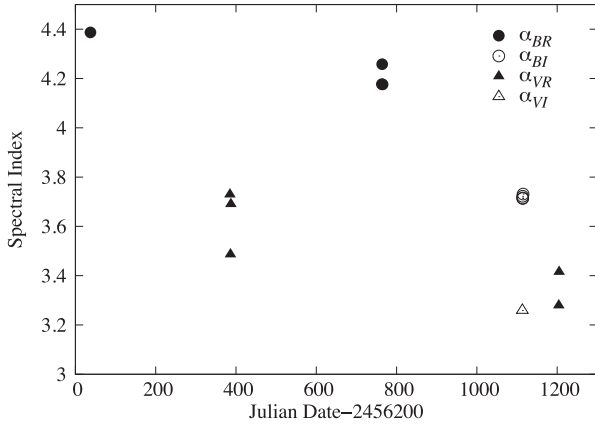
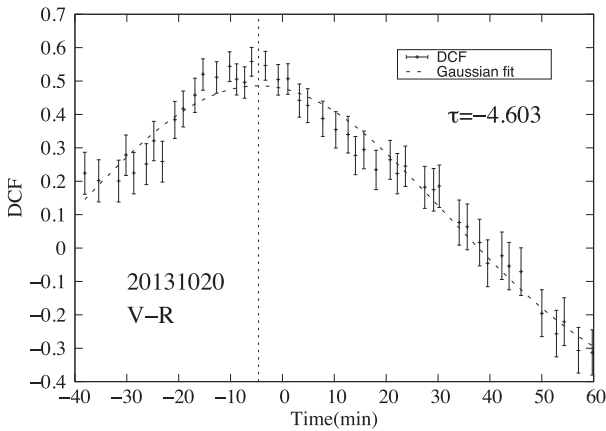
We performed the correlation analysis to search for the possible inter-band time lags by using two cross-correlation methods. The first one is the z -transformed discrete correlation functions (ZDCF) method (Alexander 1997). ZDCF deals with undersampled light curves and divides all observation points into equal bins. It uses Fisher's z -transform to stabilize the highly skewed distribution of the correlation coefficient. The Gaussian fitting (GF) is made to the central ZDCF results. Meanwhile, we try another way to measure the lags and errors by interpolated cross-correlation function method (Gaskell & Peterson 1987). The error was estimated with a model-independent Monte Carlo method, and the lag was taken as the centroid of the cross-correlation functions that were obtained with a large number of independent Monte Carlo realizations. This is the flux-randomization/random-subset selection (FR/RSS) approach described by Peterson et al. (1998, 2004). Five thousand independent Monte Carlo realizations were performed on each light curve.

One problem in the GF is that it usually underestimates the error for time delay (Wu et al. 2012). The results for ZDCF+GF are only for reference. The FR/RSS lags have significance lower than 3σ except for the $V - R$ lag on 2013 October 20. On that night, the variability in the R band led that in the V band by 11.8 min. The correlation analysis plot is displayed in Fig. 6. Date and the correlated passbands are given at left side. The peak of the Gaussian profile (the dashed line) is marked with a vertical dotted line. A negative lag (τ) means that the later variation leads the former one. Because of large errors, no time delays were found on other nights.

Table 3. Colour variations fitting results.

Date (yyyy mm dd) (1)	CI (2)	(CI) (3)	(SI) (4)	Slope (5)	Intercept (6)	r (7)	Sc (8)	Probability (9)	F (10)	F_{cri} (11)	Plot (12)
2012 11 05	$B - R$	1.785	4.387	0.048	1.767	0.298	0.475	0.010	8.081	4.351	Yes
2014 11 01	$B - R$	1.699	4.177	0.133	1.674	0.103	0.428	0.482	0.577	5.987	No
2014 11 02	$B - R$	1.732	4.258	-0.005	1.738	0.003	-0.084	0.689	0.162	3.996	No
2014 11 03	$B - R$	1.699	4.176	0.048	1.647	0.343	0.567	0.005	9.923	4.351	Yes
2015 10 18 ₁	$B - I$	2.416	3.723	0.313	2.104	0.478	0.674	0.002	13.72	4.494	Yes
2015 10 18 ₂	$B - I$	2.410	3.713	0.137	2.251	0.390	0.456	0.000	26.88	4.067	Yes
2015 10 19	$B - I$	2.422	3.732	0.179	2.225	0.917	0.956	0.000	343.3	4.149	Yes
2013 10 19	$V - R$	0.614	3.729	0.049	0.563	0.077	0.207	0.236	1.507	4.381	No
2013 10 20	$V - R$	0.574	3.486	-0.039	0.617	0.028	-0.214	0.163	1.993	3.976	No
2013 10 21	$V - R$	0.607	3.690	0.003	0.607	0.000	0.000	0.979	0.001	4.301	No
2016 01 16	$V - R$	0.539	3.280	-7.111	7.159	0.886	-0.886	0.005	30.93	6.608	Yes
2016 01 17	$V - R$	0.562	3.416	-0.561	1.112	0.479	-0.701	0.002	13.84	4.494	Yes
2015 10 17	$V - I$	1.326	3.259	0.078	1.311	0.233	0.329	0.112	3.044	4.844	No

Notes. (1) Date; (2) colour; (3) the average value of different colours (in mag); (4) the average optical spectral index; (5) colour-time correlation slope; (6) colour-time correlation intercept; (7) correlation coefficient; (8) Spearman correlation coefficient; (9) null hypothesis probability; (10) F -test results; (11) critical value of F -test; (12) if the results are greater than the critical value of F -test, we will plot them in Fig. 4.

**Figure 5.** Variation of average optical spectral index versus time covering the entire observation period for the target.**Figure 6.** ZDCF correlation and fitting result on 2013 October 20. The dashed line shows GF to the points, and the peak is marked with the vertical dotted line. τ gives the lag result.

Since the data were binned at 3-min intervals and we used the binned data to estimate the time lags, so the result of 11.8 min should be reasonable.

Wu et al. (2012) discussed the possible key factors that determine the detectability of the optical time lags. Because the V and R bands are too close, the starting time of the variations would be almost the same. It is difficult to detect the time lags between optical bands. The similar flare structures are conducive to the study of time lag detection. Maximizing the number of data points and using integration times will lead to good signal-to-noise data (Howell, Warnock & Mitchell 1988). The exposure times for this source were too short to get good signal-to-noise data in 2014. A same temporal resolution of about 1 min in different optical bands is favourable in the time-lag detection of the BL Lacertae.

4.4 Pulse analysis

Combining the results of the χ^2 test with those of the ANOVA test, we extracted five IDV light curves with several obvious flares to test the theoretical model, which was investigated by Bhatta et al. (2013). Based on the light-curve profile given by Kirk, Rieger & Mastichiadis (1998), they assumed a turbulent jet to explain the microvariability of blazar S5 0716+714.

As the strong shock hits each stochastic cell, particle acceleration and subsequent cooling by synchrotron emission produce a pulse. The convolution of these individual pulse emissions from inhomogeneous cells of various sizes and density enhancements leads to the observed microvariability.

For every local peak in the light curve, its location was taken as the centre position of the cell, its amplitude as the degree of the density enhancement and its width as its spatial extension. We applied this model to five intraday light curves by using their pulse code. We used the Doppler factor of 7.3 to calculate the pulse shape (Hovatta et al. 2009).

By varying the width and amplitude of the standard pulse, we have fitted each significant flare of the light curves. The resulting parameters for the pulses used in modelling the light curves are listed in Table 4. The first column shows the observation date, while the second column records the band. The pulse IDs and the centre time of the pulse are given in the third and fourth columns

Table 4. Pulse parameters used to fit the data.

Date (yyyy mm dd)	Band	Pulse	Centre (h)	Amp (mJy)	τ_{pulse} (h)	S_{cell} (AU)
2013 10 20	V	1	22.80	0.19	0.84	4.40
		2	24.50	0.24	1.17	6.16
		3	25.50	1.54	1.03	5.43
		4	27.38	0.24	1.39	7.34
		5	28.41	0.18	0.56	2.93
	R	1	22.64	0.23	1.11	5.87
		2	24.39	0.39	1.20	6.31
		3	25.39	1.63	1.06	5.57
		4	27.25	0.38	1.34	7.04
2014 11 03	R	1	24.00	0.05	1.36	7.19
		2	27.12	0.12	3.34	17.60
2015 10 16	I	1	22.65	0.20	0.28	1.47
		2	23.82	0.28	1.53	8.07
		3	26.00	0.35	2.48	13.06
		4	28.30	0.50	1.75	9.24
	I	5	29.90	0.60	1.67	8.80
		2	6.00	0.32	1.87	9.83

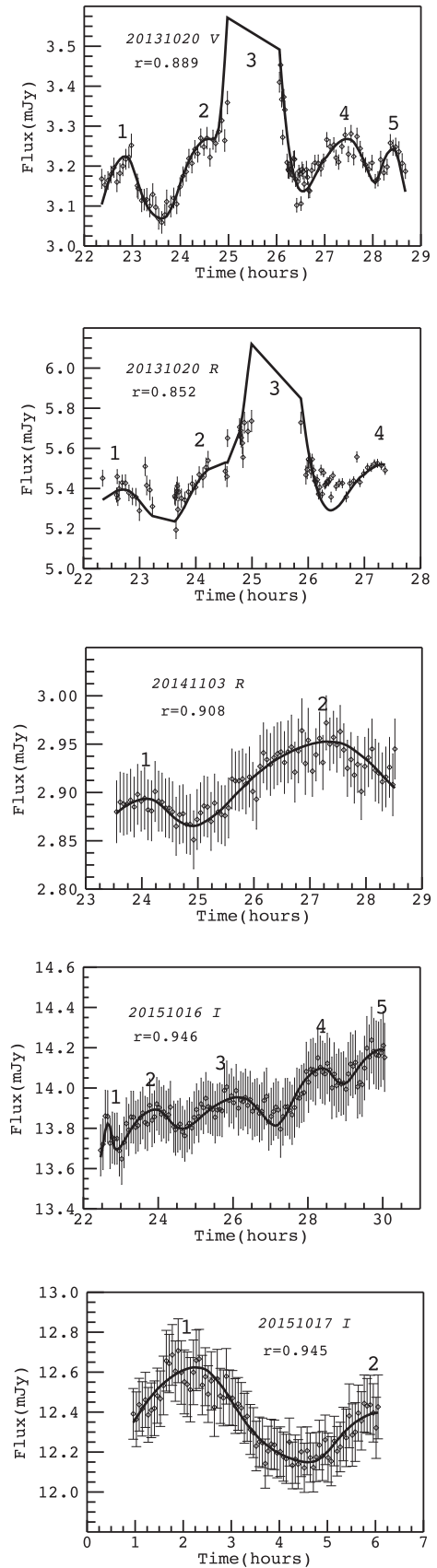
respectively, and the amplitude is in the fifth column. Column 6 gives width (τ_{pulse}) of each pulse. The number quoted in Column 7 is an estimate of the size of the cell in AU based on the assumed shock speed ($u_s = 0.1c$) and the duration of the pulse. Fig. 7 shows the light curves fitted with the convolved pulses. Although the fit is not unique, it is representative of how well the model compares to the data. The correlation coefficient (r) of each fitting is calculated. On 2013 October 20, the peaks were missing in both bands; therefore, the correlation coefficients of fitting are less than 0.9. Based on the centre time of each pulse on 2013 October 20, the time lag is estimated to be about 8 min, which is consistent with the FR/RSS result. The other 3 d are well fitted with the model.

5 DISCUSSION

There are various models to explain the IDV flux of blazars. Intrinsic ones include the instabilities in accretion disc (Wiita 1996) and the shocks travelling down the jet (e.g. Marscher 1996, 2014, and references therein). Extrinsic ones involve gravitational microlensing (Schneider & Weiss 1987) and interstellar scintillation (Bignall et al. 2003).

Following the turbulent jet model, Bhatta et al. (2013) interpreted the microvariability as emission from individual synchrotron cells, which are energized by a plane shock propagating down the jet. This results in an increase in flux resembling a pulse. Since turbulence is a stochastic process, each microvariability curve is a realization of it. We use five IDV light curves with several flares to test their theoretical model and get the turbulent parameters from our observations. There is a large range of length scales for the turbulent vortices. The largest cell size is ~ 17.6 AU that could correspond either to the correlation length scale or to the physical width of the jet, while the smallest cell size of around 1.5 AU, could correspond to the Kolmogorov scalelength of the turbulent plasma. We can get a picture of the underlying turbulent structure. Fitting pulses to BL Lacertae microvariability curves give us a much better indication of the turbulent nature of the plasma in these sources.

Miller et al. (1989) argue that the microvariations are produced very close to the central supermassive black hole (BH). Several works have attempted to estimate the mass of the BH in BL Lacertae (Fan, Xie & Bacon 1999; Woo & Urry 2002; Capetti, Raiteri

**Figure 7.** Light curves fitted with the convolutions of the synchrotron pulses of different amplitudes and widths listed in Table 4.

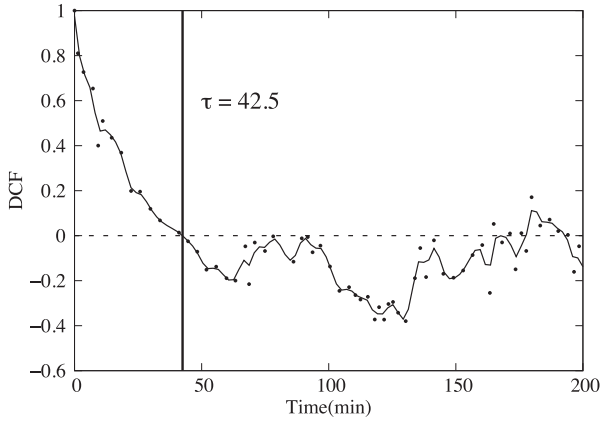


Figure 8. The ZDCF (discrete points) and its smoothed line (solid line) on 2013 October 20. τ gives the calculation of time-scale.

& Buttiglione 2010; Ghisellini et al. 2010; Gupta et al. 2012), which seems to be $0.1\text{--}6 \times 10^8 M_\odot$. If we assume fluctuations in the inner portions of the accretion disc, the observed minimum time-scale Δt_{obs} will provide an upper limit to the mass of BH. Emmanoulopoulos, McHardy & Uttley (2010) pointed out that the first-order structure function (SF) sometimes leads to incorrect claims of time-scales. Hence, we adopt the ZDCF (in autocorrelation mode) method to get a possible IDV time-scale. We choose the minimum zero-crossing time of the DCF as the correlation time-scale and get the time-scale of variability of 42.5 min on 2013 October 20 (in Fig. 8). Then according to Gupta et al. (2012), the mass of BH can be estimated by

$$M_{\text{BH}} = \frac{c^3 \Delta t_{\text{obs}}}{10G(1+z)}. \quad (7)$$

For our target, M_{BH} is calculated to be $0.49 \times 10^8 M_\odot$. If the variations arise in the jets and are not explicitly related to the inner region of the accretion disc, the BH mass estimation is invalid.

We might try to launch possible multiwavelengths observation campaign in the future to gain a much more comprehensive understanding of the physical model of blazars.

6 CONCLUSIONS

Our conclusions are summarized as follows:

- (i) We carried out a four-colour monitoring programme on BL Lacertae from 2012 to 2016. The variations were well correlated in all bands.
- (ii) The amplitude of IDV is greater in higher energy bands.
- (iii) After the host galaxy contribution was removed, the source exhibits a BWB trend. The spectral indices varied slightly.
- (iv) The possible time delays are about 10 min between variations in the V and R bands. The IDV light curves with flares are helpful to study time-lag detection. Further observations programme with a temporal resolution of about 1 min are needed to validate our time-lag results.
- (v) Our data can be well fit by the model of individual synchrotron pulses.

ACKNOWLEDGEMENTS

We would like to thank the anonymous referee for insightful comments and suggestions to improve this manuscript. This work has

been supported by the National Basic Research Programme of China 973 Programme 2013CB834900; Chinese National Natural Science Foundation grants 11273006 and U1531242; and the fundamental research funds for the central universities and Beijing Normal University.

REFERENCES

- Agarwal A., Gupta A. C., 2015, *MNRAS*, 450, 541
 Agarwal A. et al., 2016, *MNRAS*, 455, 680
 Alexander T., 1997, in Maoz D., Sternberg A., Leibowitz E. M., eds, *Astrophysics and Space Science Library*, Vol. 218, Astronomical Time Series. Kluwer, Dordrecht, p. 163
 Bach U. et al., 2006, *A&A*, 456, 105
 Bessell M. S., Castelli F., Plez B., 1998, *A&A*, 333, 231
 Bhatta G. et al., 2013, *A&A*, 558, A92
 Bignall H. E. et al., 2003, *ApJ*, 585, 653
 Böttcher M. et al., 2003, *ApJ*, 596, 847
 Capetti A., Raiteri C. M., Buttiglione S., 2010, *A&A*, 516, A59
 Carini M. T., Miller H. R., Noble J. C., Goodrich B. D., 1992, *AJ*, 104, 15
 Covino S. et al., 2015, *A&A*, 578, A68
 de Diego J. A., Dultzin-Hacyan D., Ramírez A., Benítez E., 1998, *ApJ*, 501, 69
 Emmanoulopoulos D., McHardy I. M., Uttley P., 2010, *MNRAS*, 404, 931
 Epstein E. E. et al., 1972, *ApJ*, 178, L51
 Falomo R., Pian E., Treves A., 2014, *A&AR*, 22, 73
 Fan J. H., Xie G. Z., Bacon R., 1999, *A&AS*, 136, 13
 Fiorucci M., Ciprini S., Tosti G., 2004, *A&A*, 419, 25
 Fossati G., Maraschi L., Celotti A., Comastri A., Ghisellini G., 1998, *MNRAS*, 299, 433
 Gaskell C. M., Peterson B. M., 1987, *ApJS*, 65, 1
 Gaur H., Gupta A. C., Wiita P. J., Uemura M., Itoh R., Sasada M., 2014, *ApJ*, 781, L4
 Gaur H. et al., 2015, *MNRAS*, 452, 4263
 Ghisellini G., Tavecchio F., Foschini L., Ghirlanda G., Maraschi L., Celotti A., 2010, *MNRAS*, 402, 497
 Gupta A. C., Fan J. H., Bai J. M., Wagner S. J., 2008, *AJ*, 135, 1384
 Gupta S. P., Pandey U. S., Singh K., Rani B., Pan J., Fan J. H., Gupta A. C., 2012, *New Astron.*, 17, 8
 Hagen-Thorn V. A., Larionov V. M., Jorstad S. G., Larionova E. G., 2002, *AJ*, 124, 3031
 Hagen-Thorn V. A., Larionov V. M., Jorstad S. G., Arkharov A. A., Hagen-Thorn E. I., Efimova N. V., Larionova L. V., Marscher A. P., 2008, *ApJ*, 672, 40
 Heidt J., Wagner S. J., 1996, *A&A*, 305, 42
 Hovatta T., Valtaoja E., Tornikoski M., Lähteenmäki A., 2009, *A&A*, 494, 527
 Howard E. S., Webb J. R., Pollock J. T., Stencel R. E., 2004, *AJ*, 127, 17
 Howell S. B., Warnock A., III, Mitchell K. J., 1988, *AJ*, 95, 247
 Hu S. M., Wu J. H., Zhao G., Zhou X., 2006, *MNRAS*, 373, 209
 Hyvönen T., Kotilainen J. K., Falomo R., Örn Dahl E., Pursimo T., 2007, *A&A*, 476, 723
 Ikejiri Y. et al., 2011, *PASJ*, 63, 639
 Impy C. D., Bychkov V., Tapia S., Gnedin Y., Pustilnik S., 2000, *AJ*, 119, 1542
 Kirk J. G., Rieger F. M., Mastichiadis A., 1998, *A&A*, 333, 452
 Mannucci F., Basile F., Poggianti B. M., Cimatti A., Daddi E., Pozzetti L., Vanzì L., 2001, *MNRAS*, 326, 745
 Marscher A. P., 1996, in Miller H. R., Webb J. R., Noble J. C., eds, *ASP Conf. Ser. Vol. 110, Blazar Continuum Variability*. Astron. Soc. Pac., San Francisco, p. 248
 Marscher A. P., 2014, *ApJ*, 780, 87
 Marscher A. P. et al., 2008, *Nature*, 452, 966
 Miller J. S., Hawley S. A., 1977, *ApJ*, 212, L47
 Miller H. R., Carini M. T., Goodrich B. D., 1989, *Nature*, 337, 627
 Nesci R., Maesano M., Massaro E., Montagnì F., Tosti G., Fiorucci M., 1998, *A&A*, 332, L1

- Nikolashvili M. G., Kurtanidze O. M., 2004, *Nucl. Phys. B*, 132, 205
- Papadakis I. E., Boumis P., Samaritakis V., Papamastorakis J., 2003, *A&A*, 397, 565
- Papadakis I. E., Villata M., Raiteri C. M., 2007, *A&A*, 470, 857
- Peterson B. M., Wanders I., Horne K., Collier S., Alexander T., Kaspi S., Maoz D., 1998, *PASP*, 110, 660
- Peterson B. M. et al., 2004, *ApJ*, 613, 682
- Racine R., 1970, *ApJ*, 159, L99
- Raiteri C. M. et al., 2009, *A&A*, 507, 769
- Raiteri C. M. et al., 2013, *MNRAS*, 436, 1530
- Sasada M. et al., 2008, *PASJ*, 60, L37
- Scarpa R., Urry C. M., Falomo R., Pesce J. E., Treves A., 2000, *ApJ*, 532, 740
- Schneider P., Weiss A., 1987, *A&A*, 171, 49
- Smith P. S., Balonek T. J., Heckert P. A., Elston R., Schmidt G. D., 1985, *AJ*, 90, 1184
- Sorcía M. et al., 2013, *ApJS*, 206, 11
- Speziali R., Natali G., 1998, *A&A*, 339, 382
- Stalin C. S., Gopal-Krishna Sagar R., Wiita P. J., Mohan V., Pandey A. K., 2006, *MNRAS*, 366, 1337
- Tommasi L. et al., 2001, *A&A*, 376, 51
- Urry C. M., Padovani P., 1995, *PASP*, 107, 803
- Vagnetti F., Trevese D., Nesci R., 2003, *ApJ*, 590, 123
- Villata M. et al., 2002, *A&A*, 390, 407
- Villata M. et al., 2003, in Takalo L. O., Valtaoja E., eds, *ASP Conf. Ser. Vol. 299, High Energy Blazar Astronomy*. Astron. Soc. Pac., San Francisco, p. 221
- Villata M. et al., 2004, *A&A*, 424, 497
- Wagner S. J., Witzel A., 1995, *ARA&A*, 33, 163
- Webb J. R., 2016, *Galaxies*, 4, 15
- Webb J. R. et al., 1998, *AJ*, 115, 2244
- Westfold K. C., 1959, *ApJ*, 130, 241
- Wierzhowska A., Ostrowski M., Stawarz Ł., Wagner S., Hauser M., 2015, *A&A*, 573, A69
- Wiita P. J., 1996, in Miller H. R., Webb J. R., Noble J. C., eds, *ASP Conf. Ser. Vol. 110, Blazar Continuum Variability*. Astron. Soc. Pac., San Francisco, p. 42
- Woo J.-H., Urry C. M., 2002, *ApJ*, 579, 530
- Wu J., Böttcher M., Zhou X., He X., Ma J., Jiang Z., 2012, *AJ*, 143, 108

This paper has been typeset from a $\text{\LaTeX}$ file prepared by the author.