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A Comparison of Several Image Locating Algorithms in Astronomical Instruments† ⋆

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Abstract In astronomical observations at optical wavelengths, a fast image tracking system can be adopted to reduce the effects of the atmospheric seeing and telescopic tracking error, and therefore improve the observing efficiency. Aiming at the need of astronomical observations, totally 5 kinds of algorithms in two categories were selected to make a comparative study on their accuracies and stabilities under different noise conditions by both numerical experiment and laboratory test. The results indicate that the normalized cross-correlation method and barycenter method have not only a higher accuracy but also a better reliability against interferences, they will be applied to the high-resolution spectrograph of the Xinglong 2.16 m telescope and the scientific instruments of the SONG (Stellar Observations Network Group) project, respectively.

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1. INTRODUCTION

In the scientific instruments of modern astronomical telescopes, an image tracking system can be adopted to reduce the effects of the atmospheric seeing and telescopic tracking error[1],

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and therefore increase the light flux of the system and improve the quality of scientific data. This kind of tracking/correction system makes error corrections by calculating the image shift, it requires both the high tracking accuracy and the high speed of correction. Aiming at these practical requirements, this paper made a comparative analysis on the 5 kinds algorithms in two categories. One category of algorithms obtain directly the displacement between two images by through image matching, and in which the cross-correlation algorithm[1] was selected; another category of algorithms obtain indirectly the displacement from the difference between two measured results of the light spot's image center, in which the barycenter method[2], Hough transform method[3], surface fitting method[4] and circle fitting method[5] were selected. However, each kind of algorithm has advantages and shortages of its own, it is very important to select a proper algorithm in the practical application, which affects directly the measuring accuracy and reliability of the system. In order to make analysis on the accuracy and reliability for the above-mentioned several algorithms and select an appropriate algorithm, we first added the random noises of different levels on an artificial light spot and made a theoretical analysis on the measured results of these algorithm; then we built a test system composed of the JDW3-250 helium-neon laser, high-speed camera, and one-dimensional electrically-controllable translation bench, and made verification on the results of theoretical analysis, with the laser spot image replacing the star image to be observed by an astronomical instrument. The algorithms of this paper have been applied to the optical tests in laboratory, and will be integrated in the control software of scientific instruments of relevant astronomical telescopes.

2. APPLICATIONS TO INSTRUMENTS OF ASTRONOMICAL OBSERVATIONS

The high-resolution spectrograph of the Xinglong 2.16 m telescope of National Astronomical Observatories will be equipped with a set of TIP/TILT atmospheric seeing correction system, on the refractive focal point of the SONG project telescope two sets of TIP/TILT close-loop correction systems will be installed[1], and on its Nasmyth focal point a set of focal-point tracking system will be installed. The scientific instruments on the refractive focal point of the SONG telescope include two sets of TIP/TILT correctors, which use two sets of high-speed cameras to collect respectively the stellar image of the scientific target and the pupil image of the optical system (the exit pupil image of the telescope and refractive optical system, equivalent to the imaging of the telescope primary mirror in front of the instrument), and control the corresponding TIP/TILT mechanisms via the feedback signals of two sets of high-speed cameras to realize the simultaneous corrections for the pupil shift and image drift, and therefore partially correct for the effects of the imperfect atmospheric seeing and telescopic tracking error to increase the light flux on the slit of the spectrograph. The image tracking system in the instrument is responsible for the realtime fast image acquisition, the calculation of the value of image shift, the calibrations of the value of image shift and the value of control signal, the realtime control algorithm, etc.

Considered that the performance of the TIP/TILT correction system is mainly determined by the performance of the image processing system, this paper will focus on the

[1] Anton Norup Sørensen, SONG General Optical Layout and Functions, 2010 (internal report)

selection of the algorithms for calculating the value of image shift.

3. SEVERAL COMMON-USED IMAGE TRACKING AND LOCATING ALGORITHMS

The image tracking and locating are actually to make the feedback adjustment on the system according to the value of the shift between two images, hence there are rather high requirements on the accuracy and stability of the algorithm of image displacement detection, this paper made analysis on the normalized cross-correlation method, barycenter method, Hough transform method, Gaussian fitting method, and circle fitting method. Among these algorithms, the normalized cross-correlation method can obtain directly the displacement between two images, and other 4 methods have to obtain indirectly the displacement by through the difference between the detected results of spot image centers.

3.1 Normalized Cross-correlation Method

By using the digital image processing technique, the normalized cross-correlation method measures the displacement via the detection of the correlation between the two-frame images before and after the object is displaced[1]. Assuming that $\mathbf{f}(i,j)$ is the image field of reference image, $\mathbf{g}(i+u,j+v)$ is the image field after the detecting image moves for (u,v), (i,j) represents the observational planar coordinate system, $M\times N$ represents the image size, then the normalized cross-correlation function is

$$R(u,v)=\frac{\sum_{i=1}^{M}\sum_{j=1}^{N}f(i,j)g(i+u,j+v)}{\sqrt{\sum_{i=1}^{M}\sum_{j=1}^{N}[f(i,j)]^2\sum_{i=1}^{M}\sum_{j=1}^{N}[g(i+u,j+v)]^2}}\,. \tag{1}$$

Using Eq.(1), the cross-correlation coefficients between two frames of images can be calculated, these correlation coefficients contain the information of displacement of two images, by a suitable algorithm the maximum value of correlation function can be found, and the peak position (u,v) of correlation, namely the displacement between the two frames of images, can be derived.

3.2 Barycenter Method

The barycenter method can be considered as an weighting-type method taken the grey value as the weighting factor, at first it processes the image to be the image of 256 gray levels, then it derives the spot center, the difference between the spot centers of the measuring image and reference image is the displacement between the two images, this algorithm is simple to understand and fast to calculate. Assuming that $I(i,j)$ is the grey value of the pixel on the i-th line and j-th column of the grey-leveled image, then the formula for calculating the spot center is[2]

$$x_0 = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N} iI(x,y)}{\sum_{i=1}^{M}\sum_{j=1}^{N} I(x,y)}, \ y_0 = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N} jI(x,y)}{\sum_{i=1}^{M}\sum_{j=1}^{N} I(x,y)}. \tag{2}$$

3.3 Hough Transform Method

Hough transform is a kind of transformation between the image space and the parameter space[3]. The principle for applying the Hough transform to the circle detection is as follows: assuming that the parameter equation of a circle is $r = \sqrt{(x-x_0)^2 + (y-y_0)^2}$, in which (x, y) represents the points of spot pattern in the image, (x_0, y_0) indicates the position of spot center, r is the spot radius. When the Hough transform is used to detect a circle, we have to estimate previously the possible range of the values of (x_0, y_0), and discretize the parameter space (x_0, y_0), then to have the every point of spot pattern in the image space substitute into the parameter equation, and according to the calculated result, to put successively the every point in the 3-dimensional space (x_0, y_0, r) to the vote and record by the rule of least distance, the point won most tickets is the detected result. This algorithm needs the discretization of parameter space, it restricts the measuring accuracy, the point von most tickets may be not unique, and the obtained position of spot center differs rather large with the selected point.

3.4 Gauss Fitting Method

The Gauss fitting method is to fit the distribution of light spot with a Gaussian surface, and take the extreme point of the Gauss fitting function as the central position of the light spot[4]. The 2-dimensional distribution of the light spot can be expressed as

$$I(x,y) = H\exp\left[-\frac{(x-x_0)^2}{2\sigma_x^2} - \frac{(y-y_0)^2}{2\sigma_y^2}\right]. \tag{3}$$

In Eq.(3), H is the amplitude of the Gaussian distribution, σ_x and σ_y are the standard deviations in the two directions. To make logarithm on the both sides of Eq.(3), after the squared terms are expanded and simplified into a polynomial, we have

$$z = p_4 + p_3 y + p_2 x + p_1 y^2 + p_0 x^2 . \tag{4}$$

In which, p_0, p_1, p_2, p_3, p_4 are the coefficients of the polynomial expression, to have all spot points (points with nonzero grey values) substitute into Eq.(4), and obtain all the coefficients by using the least-squares method, then the position of spot center can be obtained to be $x_0 = -p_2/2p_0$, $y_0 = -p_3/2p_1$.

3.5 Circle Fitting Method

The algorithm for locating the spot center by means of circle fitting is to approximate the spot profile with a circle according to the least-squares principle, and therefore obtain the position of spot center[5]. If **E** represents the assembly of the points on the spot profile, then the coordinates (x_0, y_0) of the spot center can be obtained from the following formula:

$$\begin{cases} x_0 = \frac{(\overline{x^2}\,\overline{x}+\overline{x}\,\overline{y^2}-\overline{x^3}-\overline{xy^2})(\overline{y}^2-\overline{y^2})-(\overline{x^2}\,\overline{y}+\overline{y}\,\overline{y^2}-\overline{x^2y}-\overline{y^3})(\bar{x}\bar{y}-\overline{xy})}{2(\overline{x}^2-\overline{x^2})(\overline{y}^2-\overline{y^2})-2(\bar{x}\bar{y}-\overline{xy})^2} \\ y_0 = \frac{(\overline{x^2}\,\overline{y}+\overline{y}\,\overline{y^2}-\overline{x^2y}-\overline{y^3})(\overline{x}^2-\overline{x^2})-(\overline{x^2}\,\overline{x}+\overline{x}\,\overline{y^2}-\overline{x^3}-\overline{xy^2})(\bar{x}\bar{y}-\overline{xy})}{2(\overline{x}^2-\overline{x^2})(\overline{y}^2-\overline{y^2})-2(\bar{x}\bar{y}-\overline{xy})^2} \end{cases}, \quad (5)$$

in which $\overline{x^m y^k} = \sum_{i \in \mathbf{E}} x_i^m y_i^k / \sum_{i \in \mathbf{E}} l$, ($m$ =0,1,2,3, k =0,1,2,3), (x_i, y_i) represents the coordinates of a point on the spot profile. The advantages of the circle fitting method are the rather high computing speed and accuracy, but it is easy to be affected by the two-valued threshold, when different threshold values are selected, the obtained center deviation may differ rather markedly.

4. ANALYSIS ON THE NOISE SENSIBILITIES OF ALGORITHMS

For the track/correction system of an astronomical telescope, in order to obtain a better effect of correction, the image acquisition speed is required to be 60 frames per second, such a relatively short exposure time causes the collected photon signal not strong enough, especially for dark stars, the exposure time should be increased to improve the signal-noise ratio, hence higher requirements are proposed for the accuracy and stability of an image tracking and locating algorithm.

For analyzing and assessing the performances of different algorithms, this paper made the noise sensibility analysis using an artificial light spot of Gaussian distribution. Particularly, a frame of ideal spot image of 494×656 pixels was produced at first, with the central position of (247, 328) and the radius of 145 pixel (the rightward and downward directions are the positive directions of the x axis and y axis), and this was taken as the reference image (Fig.1), to translate the ideal image by a known distance, we obtained two frames of images; then on one of the ideal images we added in the Gaussian white noise with the mean value μ =0 and the standard deviation σ = 0, 0.02, 0.04, $\cdots$, 0.4, respectively (Fig.2); finally, using the selected several algorithms we obtained directly or indirectly the translation for the noisy images at different noise levels, the error between the calculated translation and the actual translation indicates the performance of the algorithm. Theoretically, if the algorithm is good enough, the image noise will have no contribution to the displacement, but as the noise increases the situation will become worse.

Fig.3 shows the curve of the errors of the detected displacement in the x direction when the noises with the different standard deviations were added in, the abscissa indicates the standard deviation of the random noise. As shown in this figure, for all algorithms the detected result exhibits the up-and-down fluctuations, which represent the directions of the spot shifts caused by the uncertainty of the added noise, the greater the fluctuations, the worse the stability, in theory this should be a curve passing through the original point along the x axis. From Fig.3, we can find: the results of the Hough transform and circle fitting methods are worse, especially the detected result of the Hough transform method has occurred a large error even when the noise standard deviation is less than 0.05, this is caused by the fact that both methods make calculations on the basis of partial data on the spot profile, because of the small quantity of data, they are easy to be affected by the noise interferences and the selection of threshold values, in contrast to the other 3 kinds methods, which put whole image data into calculations, these two kinds of algorithms have a rather low stability; the Gauss fitting method has a rather high stability when the noise standard deviation is less than 0.2, but the fluctuations are apparent when the noise standard deviation is greater than 0.2; the detected results of the barycenter and normalized cross-correlation methods are rather good, when the noise standard deviation is less than 0.3, their detected results have a rather small fluctuation, and commonly concentrate around an ideal value, exhibiting a rather high stability, but the stability declines with the increasing noise standard deviation.

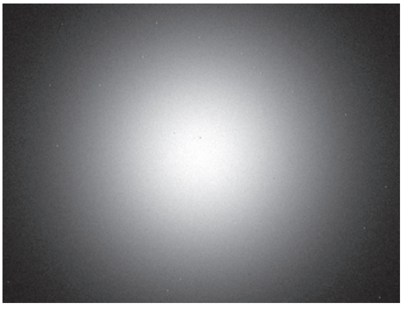

Fig. 1 Artificial light spot

Fig. 2 Random noise ($\sigma = 0.3$)

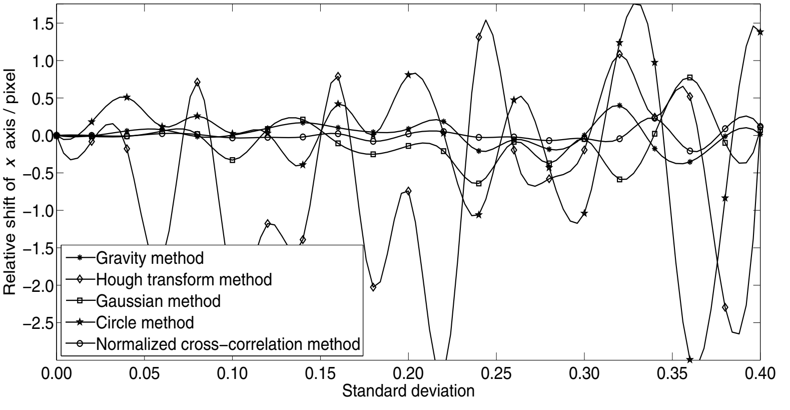

Fig. 3 Errors of the detected displacement along the x direction (μ =0)

In order to make further assessments on the several algorithms, we calculated the signal-noise ratio (SNR) of the noisy image by using the following formula[6], and analyzed the displacement detections of the every algorithm under the conditions of different signal-noise ratios,

$$SNR = 10\lg\Big[\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} s(x,y)^2 / \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} n(x,y)^2\Big]. \tag{6}$$

In Eq.(6), $s(x,y)$ is the ideal image, $n(x,y)$ is the noise signal. As mentioned above, the detected results of the Hough transform method and circle fitting method have a rather bad stability, hence here we made the analysis only on the barycenter method, Gauss fitting method and normalized cross-correlation method. Fig.4 shows the detected results on the displacement in the x direction at different SNRs, the abscissa represents SNRs. From this figure we can find that at high noise levels the measuring accuracy of the Gauss fitting method is rather low, when SRR>10 db, the error has reached 0.2 pixel; the measuring accuracy of the normalized cross-correlation method is markedly superior to the barycenter method and Gauss fitting method, even under the condition of SNR=5 db, the error is only 0.1 pixel; the accuracy of the barycenter method is also rather good, when SNR>5 db, the maximal error is within 0.2 pixel, in addition, the amount of calculations is relatively small and the computing speed is rather high, it is actually a fine algorithm if there is no very high requirement on the accuracy. Hence, summarizing the two groups of analysis, we can conclude that the normalized cross-correlation method and barycenter method have a relatively high accuracy and stability.

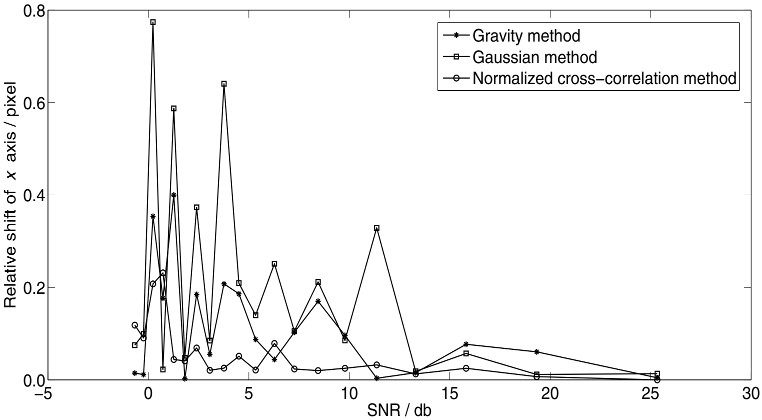

Fig. 4 Measured results of the displacement in the x direction

5. EXPERIMENTAL RESULT AND ANALYSIS

Using the JDW3-250 helium-neon laser, high-speed camera, and one-dimensional high-precision electrically-controllable translation bench (with the precision of 0.1 μm and the resolution of 0.001 μm), we built an experimental platform, and made the verification on the results of theoretical analysis, by using the collected laser spots to simulate the received light spots in astronomical observations. The experimental process was as follows: at first, to fix the CCD on the electrodriven translation bench, and adjust the height of the CCD to the same level of the laser source so that the light spot is centered on the image; secondly, to have the electric translation bench move along the horizontal direction with a certain step length, and take 10-frame pictures for each translation (Fig.5), then analyze the stability of the test result by the above-mentioned methods, and take the average of the 10-frame images as the image collected at this position; continuously make 15 translations, and adopting the image at the first position as the reference image, to calculate the relative displacements at the every position with the barycenter method and normalized cross-correlation method, respectively. For the reliability of the experimental result, each procedure was repeated for 10 times, thus 10 groups of images were obtained, it is noteworthy that the 10 groups of images correspond to the same set of positions of the translation bench. Additionally, considered the possible effect of the impurity in the air, the collected images in the experiment were not used directly for the stability analysis, before they were preprocessed by some morphological operations[7−8] and the maximum between-class variance method[8−9].

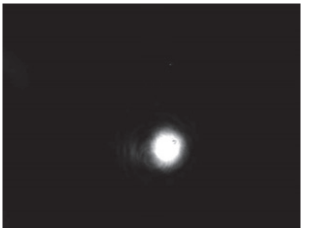

Fig. 5 The image of laser spot

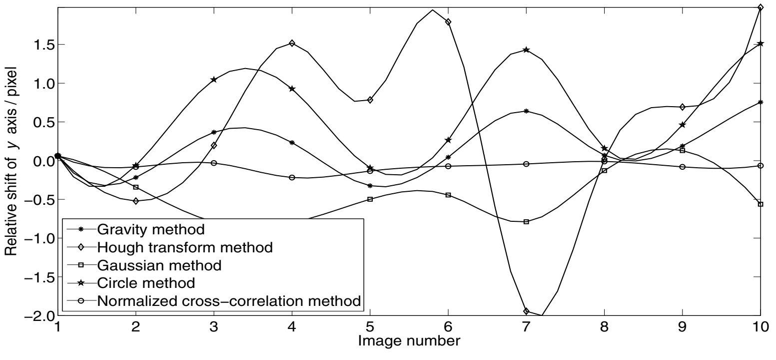

Fig. 6 The result of stability test for the displacement measurement in the y direction

Fig.6 shows the result of stability test of the 10-frame images taken at the first position of the first group, the ordinate represents the displacement in the y direction. From this figure we can find: the stability and accuracy of the Hough transform method are worst, its error is greater than 1 pixel; the stability and accuracy of the circle fitting method are also worse, this is because that both algorithms are based on the partial data of the spot profile, which are easy to be affected by the impurity of the air, the shot noise of the CCD dark current, and so on; the Gauss fitting and barycenter methods have a better test result, however, as the field distribution of the laser beam is a Gaussian distribution, but the received spot image by the astronomical instrument does not exhibit always a Gaussian distribution, so the barycenter method has a better reliability; the test result for the normalized cross-correlation method has a smaller fluctuation than other algorithms, and generally concentrates around the ideal value zero, hence it has the best accuracy and stability. Fig.7 displays the test result for the 15-frame images of the 5th group, as the images were collected in equal step lengths along the horizontal direction, the ideal result should be a straight line. From Fig.7 we can find that the displacements obtained indirectly by the barycenter method are almost coincident with the measured results of the normalized cross-correlation method, they all exhibit a linear distribution, therefore the barycenter method and normalized cross-correlation method can be used for the displacement detection, they are superior in accuracy and reliability.

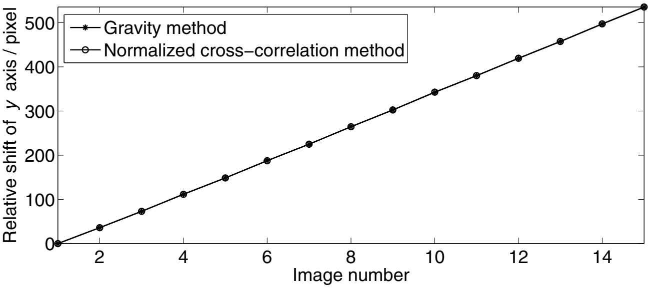

Fig. 7 The measured results of the displacements in the y direction using the 15-frame images of the 5th group

6. CONCLUSIONS

Aiming at the practical requirements on the accuracy of image positioning in the image tracking and locating systems of astronomical telescopes, a comparative analysis was made on the several algorithms. By adding in noises of different levels on an artificial light spot, we made a theoretical analysis on the errors of the algorithms, and made the verification on the results of the theoretical analysis, by using the spot images collected in the laboratory to replace the received light spots in astronomical observations. The experimental result is coincident with the theoretical analysis, both indicate that the normalized cross-correlation method and barycenter method have a relatively high accuracy and stability, satisfying the requirements on the image processing algorithms in the image tracking and locating systems of astronomical telescopes. But the two kinds of methods have their own advantages and shortages, the barycenter method has a high computing speed, but its accuracy and stability are slightly worse than the normalized cross-correlation method, in practical applications we can make a choice according to the image type and velocity requirement.

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